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AN ASYMPTOTIC INVESTIGATION OF THE DYNAMICS OF ECCENTRICALLY REINFORCED PLATES†

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With the aim of extending earlier results [1–9] obtained by the method of averaging [5–9], the equations of the vibrations of a plate reinforced by elastic ribs mounted eccentrically with respect to its median surface are investigated. A system of recurrence equations is derived, of which the first approximation corresponds to the constructive orthotropic theory. It is shown that even this approximation reveals the singularities of asymmetrically distributed ribs. Corrections for discretely situated ribs are obtained.

In dimensionless variables, the initial relations describing free vibrations of an eccentrically reinforced plate have the form

$$\left(\frac{\partial}{\partial\xi^4}+2\frac{\partial^4}{\partial\xi^2\partial\eta_1^2}+\frac{\partial}{\partial\eta_1^4}\right)w+\lambda_0 w_{\tau\tau}=0 \tag{1}$$

$$B_{01}u_{\xi\xi}+u_{\eta_1\eta_1}+(\nu B_{01}+1)v_{\xi\eta_1}=u_{\tau\tau} \tag{2}$$

$$B_{01}v_{\xi\xi}+v_{\eta_1\eta_1}+(\nu B_{01}+1)u_{\xi\eta_1}=v_{\tau\tau} \tag{3}$$

$$(w,\ w_{\eta_1},\ u,\ v)^-=(w,\ w_{\eta_1},\ u,\ v)^+ \tag{4}$$

$$w^+_{\eta_1\eta_1}-w^-_{\eta_1\eta_1}=\varepsilon\alpha_k w_{\xi\xi\eta_1} \tag{5}$$

$$w^-_{\eta_1\eta_1\eta_1}-w^+_{\eta_1\eta_1\eta_1}=\varepsilon\alpha_k w_{\xi\xi\xi\xi}+\gamma u_{\xi\xi\xi}+\lambda_1 w_{\tau\tau} \tag{6}$$

$$u^-_{\eta_1}-u^+_{\eta_1}=\beta u_{\xi\xi}-\gamma_2 u_{\tau\tau}+\lambda_1 w_{\xi\xi\xi} \tag{7}$$

$$v^-_{\eta_1}-v^+_{\eta_1}=\alpha_1 v_{\xi\xi\xi\xi}-B_{01}\lambda_2 v_{\tau\tau} \tag{8}$$

$$w=w_{\eta_1}=u=v=0 \quad \text{for} \quad \eta_1=1/2 \tag{9}$$

$$w=w_{\xi\xi}=u=v=0 \quad \text{for} \quad \xi=0,\ L=L_1/(2L_2) \tag{10}$$

$$v_\xi\sum\delta(\varphi-i)=0 \quad \text{for} \quad \xi=0,\ L \tag{11}$$

Here

$$\tau=\frac{1}{2L}\left(\frac{B_0}{\rho_1 h_0}\right)^{1/2}t,\ \lambda_0=\frac{4B_0L_2^2}{D},\ B_{01}=\frac{B}{B_0},\ B=\frac{Eh}{1-\nu^2}$$

$$\lambda_k\frac{G_0J_g}{Db},\ \gamma=\frac{E_cS}{Db},\ \beta=\frac{E_cF}{2B_0L_2},\ \lambda_1=\frac{2L_2\rho_cFB_0}{D\rho_0h},\ D=\frac{Bh^2}{12}$$

$$\lambda_2\frac{\rho_cF}{2L_2\rho_0h},\ \gamma_1=\frac{E_cS}{4B_0L_2^2},\ \alpha_1=\frac{E_cJ_1}{8BL_2^3},\ \alpha=\frac{E_cJ}{Db},\ B_0=\frac{Eh}{2(1+\nu)}$$

$$(\ldots)^{(-)}=\lim_{y\to bk_{(-)}+0}(\ldots),\ k=0\pm1,\ldots,\pm(N+1)/2$$

$$\Delta^4=\Delta^2\Delta^2,\ \Delta^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\ (\ldots)_{xy}\equiv\frac{\partial^2}{\partial x\partial y}(\ldots)$$

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ρ_0 is the density of the plate material per unit area, ρ_c is the density per unit length of the rib material, F is the cross-section area of the rib, J_g is the torque of the rib, S is the static moment of the rib relative to the median surface of the casing, J_1 is the bending moment of the rib out of the plane, J is the bending moment of the rib in the plane, w is the normal deflection, u and v are the displacements of the plate in the plane in the direction of the x and y axes, respectively, x and y are the coordinates in the plane, t is the time, E and E_c are Young's modulus of the materials of the plate and ribs, respectively, ν is Poisson's ratio of the material of the plate, G_c is the shear modulus of the rib material, h is the plate thickness, here and everywhere below the summation is from $i = -(N + 1)/2$ to $i = (N + 1)/2$, $\delta(\ldots)$ is the Dirac delta-function, N is the number of ribs, and L_1 and $2L_2$ are the lengths of the plate in the direction of the x and y axes, respectively.

We will introduce slow and fast variables η and φ, and represent the displacements in the form

$$w = w_0 + \varepsilon^4 w_1 + \varepsilon^5 w_2 + \ldots$$
$$u = u_0 + \varepsilon^2 u_1 + \varepsilon^3 u_2 + \ldots, \quad v = v_0 + \varepsilon^2 v_1 + \varepsilon^3 v_2 + \ldots \tag{12}$$

where functions of the zero approximation depend on the slow variables only.

The variability with respect to time is assumed to be small, i.e. we are considering the lower part of the spectrum. For the parameters which appear in the boundary-value problem (1)–(11) we take

$$\alpha \sim 1, \ (\gamma, \ \gamma_1, \ \lambda_0, \ \lambda_1, \ \lambda_2, \ \alpha_1, \ \alpha_k, \ \beta) \sim \varepsilon$$

Substituting series (12) into system (1)–(3) and using the new expression for the derivative

$$\frac{\partial}{\partial \eta_1} = \frac{\partial}{\partial \eta} + \varepsilon^{-1}\frac{\partial}{\partial \varphi}$$

by splitting with respect to ε we obtain a system of equations which are integrated to give

$$w = \frac{W_1}{24}\varphi^4 + C_1 + C_2\varphi + C_3\varphi^2 + C_4\varphi^3$$
$$u_1 = \frac{W_2}{2}\varphi^2 + C_5 + C_6\varphi, \quad v_1 = \frac{W_3}{2} - \varphi^2 + C_7 + C_8\varphi \tag{13}$$

The condition of matching of adjacent parts of the plate (4)–(8) can be rewritten in the form

$$(w_l, \ w_{1\eta_1}, \ u_1, \ v_1)^- = (w_1, \ w_{1\eta_1}, \ u_1, \ v_1)^+ \tag{14}$$

$$w^+_{1\varphi\varphi} - w^-_{1\varphi\varphi} = \varepsilon^{-1}\alpha_k w_{0\xi\xi\eta} \tag{15}$$

$$w^-_{1\varphi\varphi\varphi} - w^+_{1\varphi\varphi\varphi} = \alpha w_{0\xi\xi\xi\xi} + \gamma u_{0\xi\xi\xi} + \lambda_1 w_{0\tau\tau} \tag{16}$$

$$u^-_{1\varphi} - u^+_{1\varphi} = \beta u_{0\xi\xi} - \lambda_2 u_{0\tau\tau} + \gamma_1 w_{0\xi\xi\xi} \tag{17}$$

$$v^-_{1\varphi} - v^+_{1\eta} = \alpha_1 v_{0\xi\xi\xi\xi} - B_{01}^{-1}\lambda_2 v_{0\tau\tau} \tag{18}$$

Equations (16)–(18) are essentially the conditions of solvability of the equations of the first approximation, and yield at once the equations

$$W_1 + \alpha w_{0\xi\xi\xi\xi} + \gamma u_{0\xi\xi\xi} + \lambda_1 w_{0\tau\tau} = 0$$
$$W_2 + \beta u_{0\xi\xi} - \lambda_2 u_{0\tau\tau} + \gamma_1 w_{0\xi\xi\xi} = 0 \tag{19}$$
$$W_3 + [\alpha_1 v_{0\xi\xi\xi\xi}] - B_{01}^{-1}\lambda_2 v_{0\tau\tau} = 0$$

System (19) corresponds to the constructive orthotropic theory. Its main feature is that it is of order ten with respect to ζ, which raises the question of the boundary conditions. Conditions (9)–(10) become

$$w_0 = w_{0\eta} = u_0 = v_0 = 0 \quad \text{for} \quad \eta = 1/2$$
$$w_0 = w_{0\xi\xi} = u_0 = v_0 = 0 \quad \text{for} \quad \xi = 0, L$$

The missing boundary conditions are obtained by averaging relations (11)

$$v_{0\xi} = 0 \quad \text{for} \quad \xi = 0, L \tag{20}$$

For symmetrically placed ribs, system (19) splits up and the stiffness of the ribs in bending out of the plane is found to have no influence on the normal displacement w_0. Moreover, for small values of that stiffness ($\alpha_1 \sim \varepsilon^\rho$, $\rho > 1$) the term in square brackets in Eqs (19) must be omitted and the boundary condition (20) dropped. In the original variables we have

$$w_1 = -\left[\frac{\partial^4 w_0}{\partial x^4} + 2\frac{\partial^2 w_0}{\partial x^2 \partial y^2} + \frac{\partial^4 w_0}{\partial y^4} + \frac{\rho_0 h}{D}\frac{\partial^2 w_0}{\partial t^2}\right]\frac{2}{3}L_2^4 \frac{y}{b}\left(3 - 4\frac{y}{b}\right) + 2\left(\frac{y}{b}\right)^2 - \left(\frac{y}{b}\right)^3\Bigg]$$

$$u_1 = -\left[B\frac{\partial^2 u_0}{\partial x^2} + B_0\frac{\partial^2 u_0}{\partial y^2} + (\nu B + B_0)\frac{\partial^2 u_0}{\partial x \partial y} - \rho_0 h\frac{\partial^2 u_0}{\partial t^2}\right] 2L_2^2 \frac{y}{b}\left(\frac{y}{b} - 1\right)$$

The expression for v_1 is the same as u_1 with u_0 replaced by v_0.

We now construct the physical relations. By splitting the initial relations with respect to ε and averaging we obtain

$$M_1^{(0)} = D(\varkappa_1^{(0)} + \nu\varkappa_2^{(0)}) + \frac{E_c J}{b}\varkappa_1^{(0)} + \frac{E_c S}{b}\varepsilon_1^{(0)}$$

$$M_2 = D(\varkappa_2^{(0)} + \nu\varkappa_1^{(0)}), \quad M_{12}^{(0)} = D(1-\nu)\varkappa_{12}^{(0)}$$

$$T_1^{(0)} = B(\varepsilon_1^{(0)} + \nu\varepsilon_2^{(0)}) + \frac{E_c F}{b}\varepsilon_1^{(0)} + \frac{E_c S}{b}\varkappa_1^{(0)}$$

$$T_2^{(0)} = B(\varepsilon_2^{(0)} + \nu\varepsilon_1^{(0)}), \quad T_{12}^{(0)} = B_0\varepsilon_{12}^{(0)}$$

$$\varkappa_1^{(0)} = -w_{0xx}, \quad \varkappa_2^{(0)} = -w_{0yy}, \quad \varkappa_{12}^{(0)} = -w_{0xy}$$

$$\varepsilon_1^{(0)} = u_{0x}, \quad \varepsilon_2^{(0)} = v_{0y}, \quad \varepsilon_{12}^{(0)} = u_{0y} + v_{0x}$$

Here $\varkappa_1 = -w_{xx}$, $\varkappa_2 = -w_{yy}$, $\varkappa_{12} = -w_{xy}$, $\varepsilon_1 = u_x$, $\varepsilon_2 = v_y$, $\varepsilon_{12} = u_y + v_x$, M_1, M_2, M_{12} are, respectively, the bending moment and the torque, $\varkappa_1$, $\varkappa_2$ are the curvatures, $\varkappa_{12}$ is the torsion, T_1, T_2 (ε_1, ε_2) are, respectively, the forces (deformations) in the x and y directions, T_{12} is the shear force, and ε_{12} is the shear strain.

Thus, the method of averaging can be used to obtain both sequential averaged relations and corrections for discreteness in analytic form.

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